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A new method based on SOM network to generate coarse meshes for overlapping unstructured multigrid algorithm

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Abstract

A new method to generate coarse meshes for overlapping unstructured multigrid algorithm based on self-organizing map (SOM) neural network is presented in this paper. The application of SOM neural network can overcome some limitations of conventional methods and which is designed to pursuit the best structure relation between fine and coarse unstructured meshes with the object to ensure robust convergence for overlapping unstructured multigrid algorithm. Besides, this method can automate the generation of unstructured meshes and is suitable for both two and three dimensions conditions.

Keywords: SOM network; Overlapping unstructured multigrid; Optimization; Mesh generation

1. Introduction

As known, in the research field of numerical simulation in fluid dynamics, multigrid algorithms are popular methods to accelerate the convergence of numerical simulation. In order to simplify the mesh generation for complex configurations, completely unstructured meshes are often employed. So overlapping unstructured multigrid algorithm [1] is a ideal method for numerical simulation in fluid dynamics.

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For this application, firstly, a sequence of fine and coarse meshes should be generated . One suggestion is to generate fine meshes by repeatedly subdividing cells of a coarse unstructured mesh in some manner. However, generally poor topological control of the fine mesh results from such a procedure. Another widely adopted method is to generate independently all the fine and coarse meshes respectively. That is to say, a sequence of completely unrelated coarse and fine meshes are employed because the underlying theory of multigrid does not assume any relation between the various meshes, merely that variables can be transferred back and forth between them. But under some special conditions, for example, if adaptive fine mesh is needed to enhance the accuracy of result, there will be considerable difference between the structures of fine and coarse meshes generated with the method mentioned above. In the present work, a new method to generate coarse meshes based on self-organizing map (SOM) [2,4] network has been developed to ensure the structure similarity between fine and coarse meshes. So the error generated by transferring data between fine and coarse meshes will be decreased.

2. The incremental-learning SOM algorithm

The SOM consists of neurons located on a regular low-dimensional grid (usually two-dimensional (2D)), which is illustrated in Fig. 1. Each neuron i is represented by an n-dimensional prototype vector $\boldsymbol{m}_i = [m_{i0}, m_{i1}, \dots, m_{in-1}]$, where n is the dimension of the observation vector $\boldsymbol{x} = [x_0, x_1, \dots, x_{n-1}]$ which is a data sample.

On each training step, a data sample is input and the nearest unit $\boldsymbol{m}_c$ (the best matching unit) is found in the map. The prototype vectors of the best matching unit and its neighbors on the grid move towards the current sample vector:

$$m_i^{t+1} = m_i^t + h_{c(x),i}(x - m_i^t) \tag{1}$$

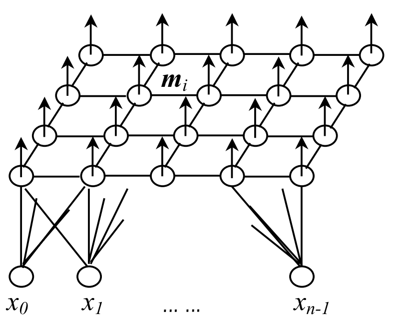

Fig. 1. The structure of SOM network.

Note t the sample index of the regression step, then index c (“winner”) is defined by the following condition:

$$\|x - m_c^t\| \leqslant \|x - m_i^t\| \quad \forall i \tag{2}$$

Here $h_{c(x),i}$ is called the neighborhood function. A simple definition of $h_{c(x),i}$ is the following: $h_{c(x),i} = \alpha(t)$ if $\|r_i - r_c\|$ is smaller than a given radius around node c (whereby this radius is also a monotonically decreasing function of t), otherwise $h_{c(x),i} = 0$. $r_i \in \Re^2$ and $r_c \in \Re^2$ are the vector locations in the display grid.

After some training steps, the SOM will arrange high-dimensional input data along its two-dimensional output space such that similar inputs are mapped onto neighboring regions of the map which means that the similarity of the input data is preserved within the representation space of the SOM.

3. The method to generate coarse meshes based on SOM

3.1. Necessity of using SOM

The traditional overlapping unstructured multigrid algorithm transfers variables between some completely unrelated meshes which are generated independently. So no measure is adopted to optimize the structure relation between fine and coarse meshes. What is important is that poor structure relations can affect the accuracy of variables transfer. Fig. 2 provides two sequences of overlapping meshes where the fine meshes of (a) and (b) are identical and the coarse meshes are different.

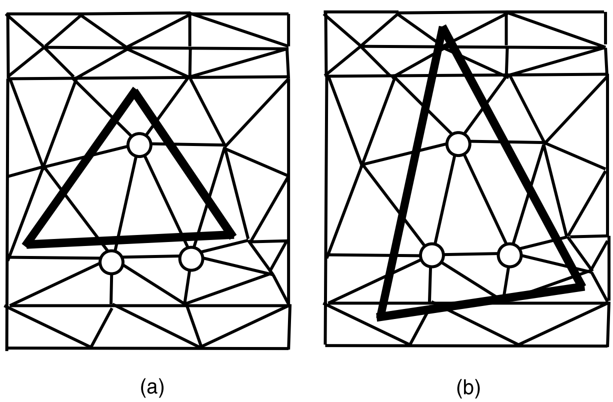

Fig. 2. Comparison between different overlapping 2D meshes.

If nodal scheme is employed, the flow variables are stored at the vertices of the triangles. When variables are transferred from the fine grid to the coarse grid, we must firstly determine in which triangle of the fine grid the expected point of the coarse grid is located, then general weighting transfer rule is used to evaluate the variables at the expected point using the variables stored at the vertices of the determined triangle of fine grid. We take an example to explain this rule. As shown in Fig. 2(a), all triangles of the fine grid with vertices located closely around or inner the triangle of the coarse grid makes a contribution to the vertices of the coarse triangle when transferring data from the fine grid to the coarse grid. In Fig. 2(b), obviously, the triangle of the fine grid with three circled point will never be considered to transfer data from the fine grid to the coarse grid. This rule results that the transfer efficiency strongly depends on the structure topology between the fine grid and the coarse grid. Dissimilarity between the points distribution of the fine grid and the coarse grid can lead to the consequence that a lot of points of the fine grid make no contributions when transferring data from the fine grid to the coarse grid. So the accuracy of data transfer will be decreased. In other words, the variables at the points of the coarse grid cannot characterize the variables at the points of the fine grid because some information of the fine grid is omitted. In this case, SOM is adopted to enhance the similarity of the points distribution of the fine grid and the coarse grid in order to ensure the accuracy of data transfer.

3.2. Application of SOM

The applications of SOM comprise two aspects, visualization and abstraction. Here, its capability of abstracting complex, nonlinear statistical relationships between high-dimensional data items is employed to optimize the distribution of the points of the coarse grid. The procedure of 2D coarse mesh generation is the following:

(1) Ensure that each layer of the meshes has the same boundary, half representative boundary points of the fine grid need to be selected to form the solid boundary and the outer boundary of the expected coarse mesh.
(2) Select randomly a quarter of the inner points of the fine mesh which will be considered as the inner points of the expected coarse mesh. The random distribution of the selected inner points will be adjusted with SOM.
(3) Applying SOM in generating the coarse mesh. The prototype vector $\boldsymbol{m}_i = [m_{i0}, m_{i1}]$ is determined with respect to the 2D coordinate vector of the ith inner point of the coarse mesh and the observation vector $\boldsymbol{x}_j = [x_{j0}, x_{j1}]$ is determined with respect to the 2D coordinate vector of the jth inner point of the fine mesh. With Eq. (1), the observation vector $\boldsymbol{x}_j$ is input one by one. Then the prototype vector $\boldsymbol{m}_i$ is adjusted towards the observation vector. This step shows that the distribution of the inner

points of the coarse mesh is adjusted to be similar with that of the fine mesh.

(4) Connect all the points of the coarse mesh using Delaunay triangulation to form the final coarse mesh after the training procedure.

3.3. Improvements for robustness and speed

Because of the random selection of the original inner coarse mesh points (step 2 shown in 3.2), the probability density of these points will be much different with that of the fine mesh. In order to enhance the robustness of the method mentioned above to generate coarse meshes and to accelerate the training procedure, a supervising method is developed in this section and described as the following:

During the training procedure, the observation vectors are inputted at random according to the probability density function. In other words, the probability density function of the fine mesh points acts as the supervising condition. For doing this, the probability density function of the fine mesh points is defined using Parzen window function [3] which is like the following:

$$\hat{p}_N(x) = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{V_N}\phi\left(\frac{x - x_i}{h_N}\right) \tag{3}$$

In the present work, $\phi((x - x_1)/h_N)$ is normal window function which is calculated as

$$\phi(u) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}u^2\right) \tag{4}$$

This supervising method is more valuable when $\hat{p}_N(x)$ varies violently with the variation of x.

4. Results

In order to show the efficiency of the new method developed in this paper, a layer of coarse mesh around a three-element airfoil is presented here. The adopted fine mesh is depicted in Fig. 3 where the density of the points varies obviously with the position in the field. According to the discussions in Section 3.1, the optimum coarse mesh should inherit the structure characteristic of the fine mesh.

Fig. 4 illustrates the original distribution of the inner coarse mesh points. Some of them are near the outer boundary and the others are near the solid boundary. Obviously, a fast convergence of overlapping multigrid algorithm

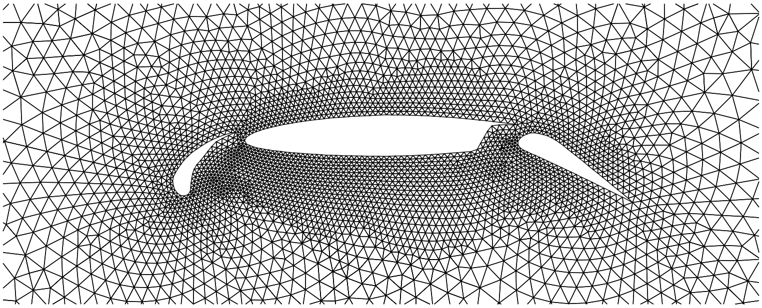

Fig. 3. The fine mesh.

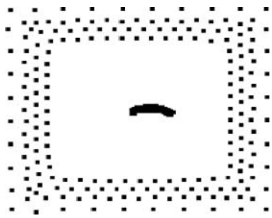

Fig. 4. The original distribution of the inner points of the coarse mesh.

will not be realized if the distribution of the coarse mesh points is not to be adjusted.

The similarity between the distributions of the fine mesh points and the coarse mesh points is embodied in Fig. 5 where (a) displays the distribution of the fine mesh points and (b) displays the final distribution of the coarse mesh points whose original positions are depicted in Fig. 4. From the comparison between the original and the final distributions of the coarse mesh points, the robustness of SOM is proved.

After the training procedure, Delaunay triangulation is employed to generate the final coarse mesh which is presented in Fig. 6. From this example, the coarse mesh generated using SOM has a similar structure with the fine mesh which will enhance the transfer accuracy, thereby the convergence for multigrid algorithm will be accelerated.

If some coarser meshes are needed for multigrid algorithm, we can generate them in the same way. Besides, in three-dimensional (3D) space, this new method is suitable too. The only difference is that the prototype vector and the observation vector of SOM are both 3D.

The following experiment further exhibits the robustness of the method developed in present work, which is displayed in Fig. 7.

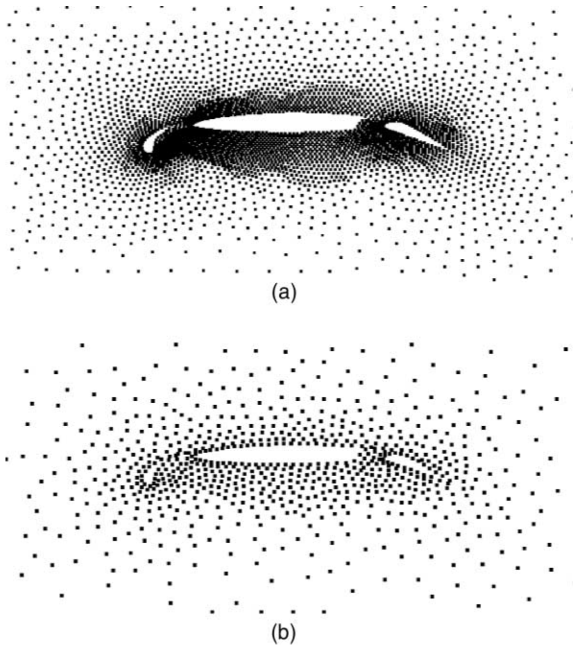

Fig. 5. Comparison between the points of the fine mesh and the coarse mesh.

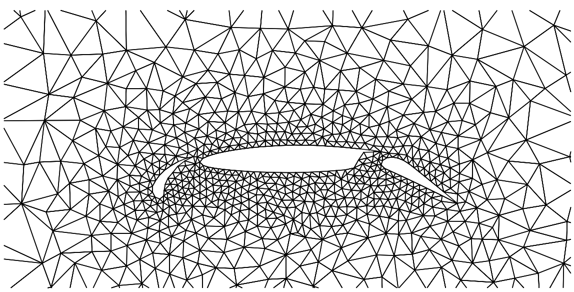

Fig. 6. The final coarse mesh.

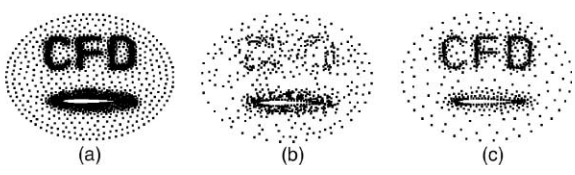

Fig. 7. Comparison between the (a) fine mesh points, (b) original coarse mesh points, and (c) final coarse mesh points.

5. Conclusions

In the present study, a new coarse mesh generation method is developed for overlapping multigrid algorithm. It is our aim to decrease the transfer error in multigrid algorithm. SOM network is employed to optimize the structure of coarse mesh with respect to the structure of a given fine mesh. The example of the mesh generation for a three-element airfoil proves that SOM could ensure that the generated coarse meshes inherit the characteristic abstracted from the fine mesh, and hence the transfer accuracy will be enhanced.

What should be emphasized particularly is that the complexity of the fine mesh will not affect the robustness of SOM because of the strong self-adaptive ability of SOM which is embodied in the example. In fact, for SOM, different fine meshes only mean different data samples, the training procedure will not be changed. Besides, although a 2D example presented, the new method developed in this paper can be easily extended to 3D space.

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