KERNEL-BASED FUZZY CLUSTERING INCORPORATING SPATIAL CONSTRAINTS FOR IMAGE SEGMENTATION

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Abstract:

The 'kernel method' has attracted great attention with the development of support vector machine (SVM) and has been studied in a general way. In this paper, we present a kernel-based fuzzy clustering algorithm that exploits the spatial contextual information in image data. The algorithm is realized by modifying the objective function in the conventional fuzzy c-means algorithm using a kernel-induced distance metric and a spatial penalty term that takes into account the influence of the neighboring pixels on the centre pixel. Experimental results on both synthetic and real MR images show that the proposed algorithm is more robust to noise than the conventional fuzzy image segmentation algorithms.

Keywords:

Image segmentation; fuzzy c-means; kernel method; magnetic resonance imaging

1 Introduction

Image segmentation plays an important role in a variety of applications such as robot vision, object recognition, and medical imaging [1]. In the last decades, fuzzy segmentation methods, especially the fuzzy c-means algorithm (FCM) [2], have been widely used in the image segmentation tasks because they can retain more information from the original image than hard segmentation methods.

Mathematically, the standard FCM objective function for partitioning a dataset $\{x_k\}_{k=1}^N$ into c clusters is given by

$$J_m = \sum_{i=1}^{c} \sum_{k=1}^{N} u_{ik}^m \| x_k - v_i \|^2 \quad (1)$$

where $\{v_i\}_{i=1}^c$ are the centers or prototypes of the clusters and the array $\{u_{ik}\}(=U)$ represents a partition matrix satisfying

$$U \in \left\{ u_{ik} \in [0,1] \left| \sum_{i=1}^{c} u_{ik} = 1, \forall k \text{ and } 0 < \sum_{k=1}^{N} u_{ik} < N, \forall i \right. \right\} \quad (2)$$

The parameter m is a weighting exponent on each fuzzy membership and determines the amount of fuzziness of the resulting classification. The FCM objective function is minimized when high membership values are assigned to pixels whose intensities are close to the centroid of its particular class, and low membership values are assigned when the pixel data is far from the centroid.

Although the original intensity-based FCM algorithm functions well on most noise-free images, it fails in segmenting images corrupted by noise and other imaging artifacts, such as the intensity inhomogeneity induced by the radio-frequency coil in magnetic resonance imaging (MRI). Many algorithms have been proposed to deal with the problem [1, 3-4]. Recently, some researchers began to incorporate spatial information into original FCM algorithm to better segment images corrupted by noise [4-6]. For example, a fuzzy rule-based system was used to impose spatial continuity on FCM [6]. And more recently, an approach was proposed for increasing the robustness of FCM to noise by directly modifying the objective function [4].

In this paper, based on the recognized power of the 'kernel method' [7, 8] in recent machine learning community, we present a kernel-based fuzzy clustering algorithm that exploits the spatial contextual information in image data. Unlike the aforementioned algorithms based on FCM for image segmentation which still adopt the squared-norm distance metric in the objective function, the proposed algorithm use a new kernel-induced metric and is shown to be more robust to noise and outlier than classical algorithms. The rest of this paper is organized as follows. In Section 2, we introduce the 'kernel method' and its application to FCM. In Section 3, the kernel-based fuzzy clustering algorithm with spatial constraints is derived. Finally, Section 4 and Section 5 give the experimental results and conclusions respectively.

2 Kernel-based fuzzy c-means

There is a trend in recent machine learning community to construct a nonlinear version of a linear algorithm using the 'kernel method ', e.g., Support Vector Machines (SVM), Kernel Fisher Discriminant (KFD) and Kernel Principal Component Analysis (KPCA) [7, 8]. The philosophy of the 'kernel method' is that every (linear) algorithm that only uses scalar products can be extended to the corresponding (nonlinear) version of this algorithm which is implicitly executed in a higher feature space through kernels. In one of our early works, a novel kernel-based fuzzy c-means algorithm is derived from this philosophy [9]. The algorithm adopts a new kernel-induced distance metric to replace the original squared-norm one in FCM.

We are in position to constructing the kernel version of the FCM algorithm and modify its objective function as following

$$J_m = \sum_{i=1}^{c}\sum_{k=1}^{N} u_{ik}^m \,\|\, \Phi(x_k) - \Phi(v_i) \,\|^2 \, . \tag{3}$$

where Φ is an implicit nonlinear map, and

$$\| \Phi(x_k) - \Phi(v_i) \|^2 = K(x_k, x_k) + K(v_i, v_i) - 2K(x_k, v_i) \tag{4}$$

Where $K(x, y) = \Phi(x)^T \Phi(y)$ is an inner product kernel function. If we adopt the Gaussian RBF kernel, i.e., $K(x, y) = \exp(-\| x - y \|^2 / \sigma^2)$, then $K(x, x) = 1$, according to Eqs. (4), Eqs. (3) can be simplified as

$$J_m = 2\sum_{i=1}^{c}\sum_{k=1}^{N} u_{ik}^m \left(1 - K(x_k, v_i)\right) \tag{5}$$

By a similar optimization way to the standard FCM algorithm, the objective function J_m can be minimized under the constraint of *U*. Specifically, taking the first derivatives of J_m with respect to u_{ik} and v_i, and zeroing them respectively, two necessary but not sufficient conditions for J_m to be at local extrema will be obtained as the following

$$u_{ik} = \frac{\left(1 - K(x_k, v_i)\right)^{-1/(m-1)}}{\sum_{j=1}^{c}\left(1 - K(x_k, v_j)\right)^{-1/(m-1)}} \tag{6}$$

$$v_i = \frac{\sum_{k=1}^{n} u_{ik}^m K(x_k, v_i) x_k}{\sum_{k=1}^{n} u_{ik}^m K(x_k, v_i)} \tag{7}$$

Here we only use the Gaussian RBF kernel for simplicity. For other kernel functions, the corresponding updating equations are a little more complex, because their derivatives are not as simple as the Gaussian RBF kernel function. It is shown that the new algorithm is more robust than the FCM under outliers and noise according to robust statistics [9].

3 Kernel-based fuzzy clustering with spatial constraints

In [4], a spatial FCM was proposed to increase the robustness to noise. Similarly, we propose a modification to Eqs. (5) by introducing a penalty term containing spatial neighborhood information. As mentioned before, this penalty term acts as a regularizer and biases the solution toward piecewise-homogeneous labeling. Such regularization is helpful in segmenting images corrupted by noise. The modified objective function is given by

$$\begin{aligned} J_m = {} & \sum_{i=1}^{c}\sum_{k=1}^{N} u_{ik}^m \left(1 - K(x_k, v_i)\right) \\ & + \frac{\alpha}{N_R}\sum_{i=1}^{c}\sum_{k=1}^{N} u_{ik}^m \sum_{r \in N_k}\left(1 - K(x_r, v_i)\right) \end{aligned} \tag{8}$$

where N_k stands for the set of neighbors that exist in a window around x_k and N_R is the cardinality of N_k. The parameter α controls the effect of the penalty term. The relative importance of the regularizing term is inversely proportional to the signal-to-noise (SNR) ratio of the image. In other words, Lower SNR would require a higher value of the parameter α, and vice versa.

An iterative algorithm of minimizing Eqs. (8) can be derived by evaluating the centroids and membership functions that satisfy a zero gradient condition. A necessary condition for Eqs. (8) to be at a local minimum is

$$u_{ik} = \frac{\left(\left(1 - K(x_k, v_i)\right) + \frac{\alpha}{N_R}\sum_{r \in N_k}(1 - K(x_r, v_i))\right)^{-1/(m-1)}}{\sum_{j=1}^{c}\left(\left(1 - K(x_k, v_j)\right) + \frac{\alpha}{N_R}\sum_{r \in N_k}(1 - K(x_r, v_j))\right)^{-1/(m-1)}} \tag{9}$$

$$v_i = \frac{\sum_{k=1}^{n} u_{ik}^m \left(K(x_k, v_i) x_k + \frac{\alpha}{N_R}\sum_{r \in N_k}\left(K(x_r, v_i) x_r\right)\right)}{\sum_{k=1}^{n} u_{ik}^m \left(K(x_k, v_i) + \frac{\alpha}{N_R}\sum_{r \in N_k} K(x_r, v_i)\right)} \tag{10}$$

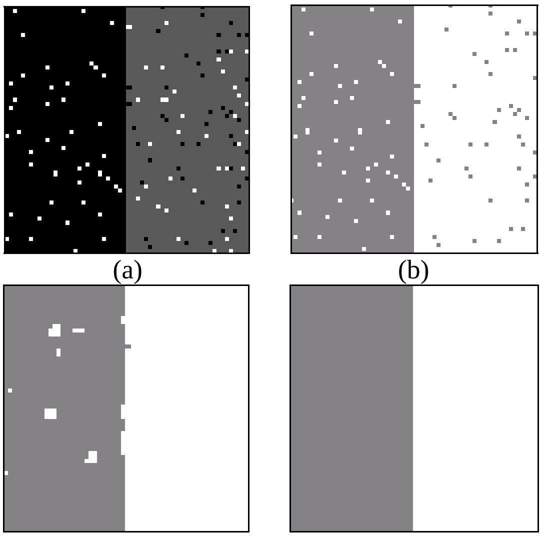

Figure 1 Comparison of segmentation results on a synthetic image with 5% 'salt & pepper' noise. (a) Original image. (b) By FCM. (c) By spatial FCM [4]. (d) Result of the proposed algorithm.

The proposed kernel-based fuzzy clustering algorithm incorporating spatial constraints can be summarized in the following steps:

Step 1) Select initial class prototypes (centroids). Set $\varepsilon > 0$ for a very small value.
Step 2) Update the partition matrix using Eqs. (9).
Step 3) Update the centroids using Eqs. (10).

Repeate Steps 2)-3) until the following termination criterion is satisfied:

$$\| V_{new} - V_{old} \| < \varepsilon$$

where $\| \cdot \|$ stand for the Euclidean norm, and V is the vector of cluster centroids.

4. Experimental results

Fig. 1 displays the results of FCM, spatial FCM [4] and the proposed algorithm when applied to a synthetic test image. The image possesses two classes with intensity values 0 and 90, and the image size is 64x64 pixels. Fig. 1(a) is the original image corrupted by 5% 'Salt & Pepper' noise and Fig. 1(b)-(d) are the results by FCM, spatial FCM and the proposed algorithm respectively. The parameters used are $\alpha = 10$, $N_R = 8$. The Gaussian RBF kernel is used in our algorithm and the kernel width is $\sigma = 150$. These values

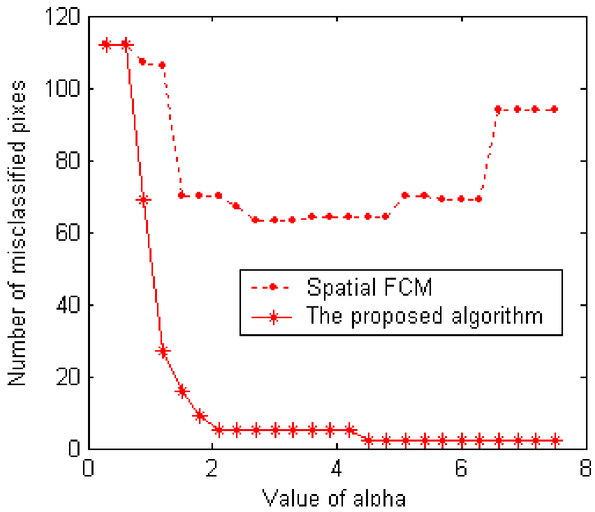

Figure 2 Comparison of classification errors on Figure. 1(a) at different values of α .are used throughout this section except for the value of α. Figure. 2 shows the comparison of classification errors of spatial FCM and our algorithm on Fig. 1(a) at different values of α. Clearly, the proposed algorithm is more robust to the presence of noise.

Figure. 3 presents a comparison of segmentation results among spatial FCM and the proposed algorithm when applied on a real MR image corrupted by 5% 'salt & pepper' noise. Figure. 3(a)-(b) are the original corrupted images, Figure. 3(c)-(d) are the results using spatial FCM, and Figure. 3(e)-(f) are the results with our algorithm respectively. The parameter $\alpha = 3.8$ is used in this experiment. Obviously, the proposed algorithm is more robust to noise than spatial FCM.

5. Conclusions

In this paper, a new approach for segmenting MR image corrupted by noise was presented. The algorithm is based on the well-known 'kernel method', and is realized by modifying the objective function in the conventional fuzzy c-means algorithm using a kernel-induced distance metric and a spatial penalty term that takes into account the influence of the neighboring pixels on the centre pixel. The proposed algorithm is shown more robust to 'salt & pepper' noise than conventional fuzzy clustering algorithms in segmenting MR images.

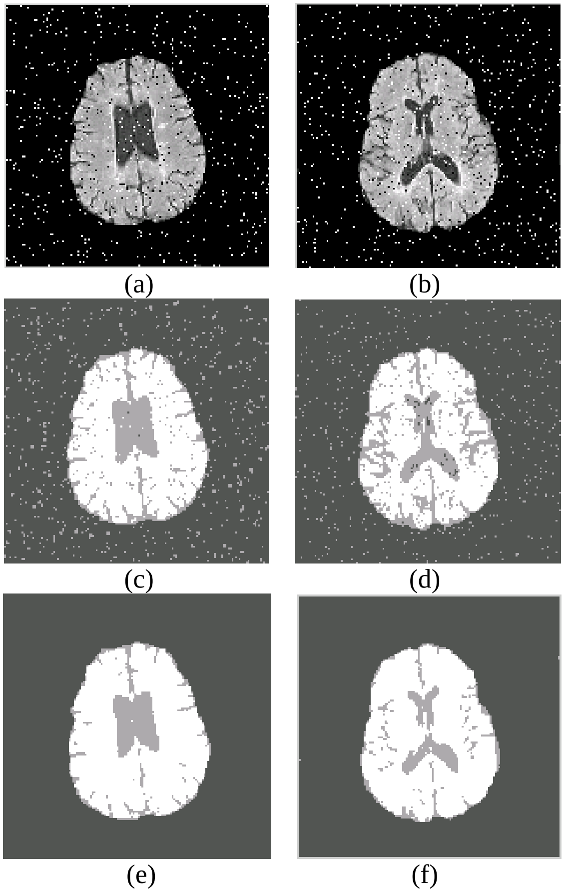

Figure3 Comparison of segmentation results on real MR images corrupted by 5% 'salt & pepper' noise. (a)-(b) Original images. (c)-(d) By spatial FCM [4]. (e)-(f) Results of the proposed algorithm.

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