Abstract—In this paper, we investigate the problem of learning feature representation from unlabeled data using a single-layer K-means network. A K-means network maps the input data into a feature representation by finding the nearest centroid for each input point, which has attracted researchers’ great attention recently due to its simplicity, effectiveness, and scalability. However, one drawback of this feature mapping is that it tends to be unreliable when the training data contains noise. To address this issue, we propose a SVDD based feature learning algorithm that describes the density and distribution of each cluster from K-means with an SVDD ball for more robust feature representation.

For this purpose, we present a new SVDD algorithm called C-SVDD that centers the SVDD ball towards the mode of local density of each cluster, and we show that the objective of C-SVDD can be solved very efficiently as a linear programming problem. Additionally, previous single-layer networks favor a large number of centroids but a crude pooling size, resulting in a representation that highlights the global aspects of the object. Here we explore an alternative network architecture with much smaller number of nodes but with much finer pooling size, hence emphasizing the local details of the object. The architecture is also extended with multiple receptive field scales and multiple pooling sizes. Extensive experiments on several popular object recognition benchmarks, such as MINST, NORB, CIFAR-10 and STL-10, show that the proposed C-SVDDNet method yields comparable or better performance than that of the previous state of the art methods.

I. INTRODUCTION

Learning good feature representation from unlabeled data is the key to make progress in recognition and classification tasks, and has attracted great attention and interest from both academia and industry recently. A representative method for this is the deep learning (DL) approach [1] with its goal to learn multiple layers of abstract representations from data. Among others, one typical DL method is the so called convolutional neural network (ConvNet), which consists of multiple trainable stages stacked on top of each other, followed by a supervised classifier [2] [3]. Many variations of ConvNet network have been proposed as well for different vision tasks [4] [5] [6] with great success.

In these methods layers of representation are usually obtained by greedily training one layer at a time on the lower level [5] [7] [3], using an unsupervised learning algorithm. Hence the performance of single-layer learning has a big effect on the final representation. Neural network based single-layer methods, such as autoencoder [8] and RBM (Restricted Boltzmann Machine, [9]), are widely used for this but they usually have many parameters to adjust, which is very time-consuming in practice.

That motivates more simple and more efficient methods for single-layer feature learning. Among others K-means clustering algorithm is a commonly used unsupervised learning method, which maps the input data into a feature representation simply by associating each data point to its nearest cluster center. There is only one parameter involved in the K-means based method, i.e., the number of clusters, hence the model is very easy to use in practice. Coates et al. [10] shows that the K-means based feature learning network is capable to achieve superior performance compared to sparse autoencoder, sparse RBM and GMM (Gaussian Mixture Model). However, the K-means based feature representation may be too terse, and does not take the non-uniform distribution of cluster size into account - Intuitively, clusters containing more data are likely to be part of the features with higher influential power, compared to the smaller ones.

In this paper, we proposed a SVDD (Support Vector Data Description, [11], [12]) based method to address these issues. The key idea of our method is to use SVDD to measure the density of each cluster resulted from K-means clustering, based on which more robust feature representation is built. Actually the K-means algorithm lacks a robust definition of the size of its clusters, since the nearest center principle is not robust against the noise or outliers commonly encountered in real world applications. We advocate that the SVDD could be a good way to address this issue. Actually SVDD is a widely used tool to find a minimal closed spherical boundary to describe the data belonging to the target class and therefore, given a cluster of data, we are expecting SVDD to generate a ball containing the normal data except outliers. Performing this procedure on all the clusters of K-means, we will finally obtain $K$ SVDD balls on which our representation can be built. In addition, to take the cluster size into account, we use the distance from the data to each ball’s surface instead of the center as the feature.

One possible problem of this method, however, may come from the instability of SVDD’s center, due to the fact that its position is mainly determined by the support vectors on the boundary and the noise in the data may deviate the center far from the mode (c.f., Fig. 5 (left)). Hence the resulting SVDD ball may not be consistent with the data’s distribution when used for feature representation. To address this issue, we add a new constraint to the original SVDD objective function to make the model align better with the data. In addition, we show that our modified SVDD can be solved very efficiently as a linear programming problem, instead of as a quadratic programming problem.

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one. Usually we need to compute hundreds of clusters, and a linear programming solution can thus save us large amounts of time. The proposed method is further extended by adopting a set of receptive fields with different sizes to capture multi-scale information ranging from detailed edge-like features to part-level features.

One of the major conclusions of Coates et al.’s series of controlled experiments on single layer unsupervised feature learning networks [10] is that compared to the choice of particular learning algorithm, the parameters that define the feature extraction pipeline, especially the number of features, have much more deep impact on the performance. Using a K-means network with 4000 features, for example, they are able to achieve surprisingly good performance on several benchmark datasets - even better than those with much deeper architectures (e.g., Convolutional Deep Belief Nets (CDBN) [13], Deep Boltzmann Machine [14] and Sparse Auto-encoder [10]). One major reason for this is that the existence of large number of basis vectors increases the chances that an input can be encoded properly by the network [10]. However, one drawback accompanying this large network is that a very crude pooling size has to be adopted (e.g., $46 \times 46$ over $92 \times 92$ feature maps) to condense the resulting feature maps, otherwise the dimensionality of the final feature representation could be prohibitively high. For example, a $3 \times 3$ pooling over 4000 feature maps with $92 \times 92$ in size would lead to a total number of features over 3.8M.

While the Coates et al.’s large single-layer network can be thought of as highlighting the benefits of learning a representation that encodes global information of an object using large number of centroids, here we explore alternative architecture of single-layer networks with much smaller number of nodes and with much finer pooling size, hence emphasizing the local details of the object in the representation. To compensate for loss of information due to the use of a smaller number of basis vectors, it is crucial to encoding the pooling response effectively. Here we use a variant of SIFT-based encoder [15], which essentially projects the responses of a pooling operation into a low dimensional space while suppressing the noise and improving the invariant properties of the final feature representation. We further extend the network with multiple receptive field scales and multiple pooling sizes for better feature learning. The feasibility and effectiveness of the proposed C-SVDD-based small single-layer network (called C-SVDDNet) is verified extensively on several object recognition benchmarks with competitive performance. A preliminary version of this work appeared in [16].

The remaining parts of this paper are organized as follows: In Section II, preliminaries are provided regarding the K-means based feature representation, then we detail our improved feature learning network in Section III. In Section IV, we investigate the performance of our network empirically over one face recognition datasets and several popular object classification datasets. We conclude this paper in Section V.

II. PRELIMINARIES

A. Unsupervised Feature Learning

The goal of unsupervised feature learning is to automatically discover useful hidden patterns/features in large datasets without relying on a supervisory signal, and those learnt patterns can be utilized to create representations that facilitate subsequent supervised learning (e.g., object classification). A typical pipeline for unsupervised feature learning includes four steps. The first step is to train a set of local filters from the unlabeled training data. Then given an input image, we construct a set of feature maps for it using the learnt filters (e.g., through an image convolution) in the second step. After applying a pooling operation on them in the third step, these feature maps are finally combined into a vector as the feature representation for the input image.

The local filters are a set of latent patterns found in the data and for this there exist many basic algorithms (e.g., K-means, Gaussian Mixture Models (GMM), autoencoder [8], RBM [9], sparse coding) and their more sophisticated variants, such as Sparse Auto-encoder [10], Sparse RBM [10], Locally-connected Neural Pyramid (LCNP) [17], Conv. Sparse Factor Analysis (CSFA) [18], Hierarchical Matching Pursuit (HMP) [19], Hierarchical Sparse Coding (HSC) [20], Fisher vector [21], and so on. The local filters learnt by these algorithms essentially contain important prior knowledge about the distribution of the data, which plays a critical role in the subsequent feature encoding. Interestingly, Coates et al. [10] shows through a series of controlled experiments that compared to how to obtain these filters, how many of them (the number of features) and how to use them (feature encoding) has more fundamental impact on the final performance of a network. In this work, we focus on the single layer K-means network, due to its simplicity, effectiveness, scalability, and its potential to serve as a good prototype to study the feature learning approaches.

B. K-means for Feature Mapping

K-means is a data clustering algorithm that divides data into a set of K clusters, with Euclidean distance as similarity measure. It aims to minimize the sum of distance between all data to their corresponding centers. Let $X = \{x_i\}, i = 1, ..., n$ be the set of $n$ d-dimensional points, $C = \{C_k\}, k = 1, ..., K$ be the set of $K$ clusters, with $c_k$ the mean of cluster $C_k$. The K-means algorithm finds the clusters by solving the following objective function, $J(C) = \sum_{k=1}^{K} \sum_{x \in C_k} \|x - c_k\|^2$.

After learning the clusters, they would be used to produce a feature mapping. So if we have $K$ clusters, the dimension of the resulting feature representation will be $K$. The simplest way for feature mapping is the so-called “hard coding” method, i.e., setting the winner cluster center on while all the others off, as follows,

$$f_k(x) = \begin{cases} 1 & \text{if } k = \text{argmin}_j \|c_j - x\|^2_2 \\ 0 & \text{otherwise} \end{cases}$$  

(1)

The resulting K-dimensional vector $f$ can be interpreted as the MAP estimate of the input point $x$ given the K-means model. However it is too sparse and is often not representative
of the full posterior mass. A better summary is the following “triangle” encoding [10]:

\[ f_k(x) = \max \{0, \mu(z) - z_k(x)\} \quad (2) \]

where \( z_k(x) = \|x - c_k\|_2 \), and \( \mu(z) \) is the mean of the elements of \( z \). This activation function outputs 0 for the feature \( f_k \) that has an above average distance to the centroid \( c_k \). This model leads to a less sparse representation (roughly half of the features could be set to be 0). Note that this “triangle” encoding strategy essentially allows us to learn a distributed representation using the simple K-means method instead of more complicated network-based methods (e.g., autoencoder and RBM), hence saving much time in training. Coates et al. [10] shows that this strategy actually leads to comparable performance to, if not better than, those based on network methods.

However, this method does not take the characteristics of each cluster into consideration. Actually, the number of data points in each cluster is usually different, so is the distribution of data points in each cluster. We believe that these differences would make a difference in feature representation as well. Unfortunately the aforementioned K-means feature mapping scheme completely ignores these and only uses the position of center for feature encoding. As shown in Fig. 1, although the data point \( x \) has the same distance to the centers \( c_1 \) and \( c_2 \) of two clusters, it should be assigned a different score on \( C_1 \) than on \( C_2 \) since the former cluster \( C_1 \) is much bigger than the latter. In practice such unequal clusters are not uncommon and the K-means method by itself can not reliably grasp the size of its clusters due to the existence of outliers. To this end, we propose an SVDD based method to describe the density and distribution of each cluster and use this for more robust feature representation.

III. THE PROPOSED METHOD

In this section, after presenting the proposed architecture of single-layer network, we give the details of the proposed Centered-SVDD method for feature encoding, and compare it with the K-means “triangle” encoding method. Then we describe our SIFT-based post-pooling layer and discuss how to extend the method to extract multi-scale information.

A. The architecture of the Single-layer Network

A typical single-layer network contains several components: an input image is first mapped into a set of feature maps using filter banks (or dictionary), which are then subjected to a pooling/subsampling operation to condense the information contained in the feature maps. Finally, the pooled feature maps are concatenated to a feature vector, which serves as the representation for the subsequent classification/cluster tasks. There are several design options in this procedure, where the size of filter bank and that of the pooling grids are the major tradeoff one has to make. Fig. 2 gives two typical schemes of network architectures, where scheme A uses a big filter bank but a crude pooling size, while scheme B has a smaller filter bank but a fine-grided pooling size.

Generally speaking, bigger filter banks help each sample find its nearby representative points more accurately but at the cost of yielding a high-dimensional representation, hence a crude pooling/subsampling is needed to reduce the dimensionality. Overall this type of architecture emphasizes more on the global aspects of the samples than on the local ones (e.g., local texture, local shape, etc.). Actually, Coates et al. show that this kind of network is able to yield state of the art results on several challenging datasets [10]. On the other hand, other works use smaller filter banks but highlight the importance of detailed local information in constructing the representation, usually based on some complicated feature encoding strategy, as done in PCANet [22] or Fisher Vector [23].

In this work, we follow the second design choice, based on the consideration that for the task of object classification, the learned representation should preserve enough local spatial information for the subsequent processing. Fig. 3 gives the architecture of our single layer network. Compared to [10], we use an improved feature encoding method named C-SVDD (detailed in the next section) and adopt the architecture of relatively small dictionary. Different to [22] or [23], we learn filter banks for feature encoding but add a SIFT-based post-pooling processing procedure onto the network, which essentially projects the responses of a pooling operation into a more compact and robust representation space.
B. Using SVDD Ball to Cover Unequal Clusters

Assume that a dataset contains \( N \) data objects, \( \{x_i\}, i = 1, \ldots, n \) and a ball is described by its center \( a \) and the radius \( R \). The goal of SVDD (Support Vector Data Description, [11]) is to find a closed spherical boundary around the given data points. In order to avoid the influence of outliers, SVDD actually faces the tradeoff between two conflicting goals, i.e., minimizing the radius while covering as many data points as possible. This can be formulated as the following objective,

\[
\min_{a, R, \xi} \quad R^2 + \lambda \sum_{i=1}^{N} \xi_i \\
\text{s.t.} \quad \|x_i - a\|^2 \leq R^2 + \xi_i \\
\xi_i \geq 0,
\]

where the slack variable \( \xi \) represents the penalty related with the deviation of the \( i \)-th training data point outside the ball, and \( \lambda \) is a user defined parameter controlling the degree of regularization imposed on the objective. With the KKT conditions, we have \( a = \sum_{i=1}^{N} x_i \), i.e., the center \( a \) of the ball is a linear combination of the data \( x_i \). The dual function of Eq. (3) is

\[
\max \sum_i \alpha_i \langle x_i, x_i \rangle - \sum_i \sum_j \alpha_i \alpha_j \langle x_i, x_j \rangle \\
\text{s.t.} \quad \sum_i \alpha_i = 1, \quad \alpha_i \in [0, \lambda], \quad i = 1, \ldots, N,
\]

where \( \alpha_i \) and \( \alpha_j \) are Lagrangian multipliers. By solving the quadratic programming problem we can get the center \( a \) and the radius \( R \).

The SVDD method can be understood as a type of one-class SVM and its boundary is solely determined by support vectors points. SVDD allows us to summarize a group of data points in a nice and robust way. Hence it is natural to use SVDD ball to model each cluster from K-means, thereby combining the strength of both models. In particular, for a given data point we first compute its distance \( h_k \) to the surface of each SVDD ball \( C_i \), and then use the following modified “triangle” encoding method for feature representation (c.f., Eq.( 2)),

\[
f_k(x) = \max\{0, g(h) - h_k(x)\},
\]

where \( h_k(x) = \|x - R_k\|_2 \) is the distance from the point \( x \) to the surface of the \( k \)-th SVDD ball, while \( g(h) \) is the average of the values \( h_k \).

C. The C-SVDD Model

Although SVDD ball provides a robust way to describe the cluster of data, one unwelcome property of the ball is that it may not align well with the distribution of data points in that cluster. As illustrated in Fig. 5 (left), although the SVDD ball...
covers the cluster \( C_1 \) well, its center is biased to the region with low density. This should be avoided since it actually gives suboptimal estimates on the distribution of the cluster of data.

To address this issue, inspired by the observation that the centers of K-means are always located at the corresponding mode of their local density, we propose to shift the SVDD ball to the centroid of the data such that it may fit better with the distribution of the data in a cluster. Our new objective function is then formulated as

\[
\begin{align*}
\min_{R, \xi, \beta} & \quad R^2 + \lambda \sum_{i=1}^{N} \xi_i \\
\text{s.t.} & \quad \|x_i - a\|^2 \leq R^2 + \xi_i \\
& \quad a = \frac{1}{N} \sum_{i=1}^{N} x_i \\
& \quad \xi_i \geq 0,
\end{align*}
\]

and its Lagrange function is as follows,

\[
\mathcal{L}(R, \xi, \alpha, \beta) = R^2 + \lambda \sum_{i=1}^{N} \xi_i + \sum_{i=1}^{N} \alpha_i \left( \|x_i - a\|^2 - R^2 - \xi_i \right) - \sum_{i=1}^{N} \beta_i \xi_i,
\]

where \( \alpha_i \geq 0 \) and \( \beta_i \geq 0 \) are the corresponding Lagrange multipliers. According to KKT Conditions, we have,

\[
\frac{\partial \mathcal{L}}{\partial R} = 2R - 2R \sum_{i=1}^{N} \alpha_i = 0, \quad \sum_{i=1}^{N} \alpha_i = 1
\]

\[
\frac{\partial \mathcal{L}}{\partial \xi_i} = \lambda - \alpha_i - \beta_i = 0
\]

Taking Eq.(8) and Eq.(9) into the Lagrange function (7) we get that

\[
\mathcal{L}(R, \xi, \alpha, \beta) = \sum_{i=1}^{N} \alpha_i \left( \|x_i - a\|^2 \right).
\]

Recalling that \( a = \frac{1}{N} \sum_{i=1}^{N} x_i \), one has the following dual function,

\[
\max \sum_{i} \alpha_i \langle x_i, x_i \rangle - \frac{2}{N} \sum_{i} \sum_{j} \alpha_i \langle x_i, x_j \rangle \\
\text{s.t.} \quad \sum_{i} \alpha_i = 1, \quad \alpha_i \in [0, \lambda], \quad i = 1, ..., N.
\]

This can be reformulated as

\[
\min \frac{2}{N} \alpha^T H e - \alpha^T F \\
\text{s.t.} \quad \alpha_i^T e = 1, \quad \alpha_i \in [0, \lambda], \quad i = 1, ..., N,
\]

where \( H = ((x_i, x_i))_{N \times N} \), \( F = ((x_i, x_i))_{N \times 1} \), \( e = (1, 1, ..., 1)^T \). This objective function is linear to \( \alpha \), and thus can be solved efficiently with a linear programming algorithm.

Since the model is centered towards the mode of the distribution of the data points in a cluster, we named our method as C-SVDD (centered-SVDD). Fig. 5 shows the difference between SVDD and C-SVDD, where the left is from SVDD and the right from C-SVDD. We can see that our new model aligns better with the density of the data points, as expected. It is also worth mentioning that the normalization parameter \( \lambda \) plays an important role in our model - a larger \( \lambda \) value would allow more noise to enter the ball, while \( \lambda = 0 \), the C-SVDD model actually reduces to the naive single-cluster K-means. More discussions on setting this value empirically will be given in Section IV.

After the model is trained, we use the modified “triangle” encoding (Eq. 5) for feature encoding, with almost the same computational complexity with its K-means counterpart.

\D. \textbf{K-means Encoding vs. C-SVDD Encoding}

To this end, it will be useful to take a brief discussion on the difference of two kinds of feature maps, i.e., K-means-based “triangle” encoding (Eq. 2) and our C-SVDD-based one. For this a pilot experiment is conducted. Particularly, we learn a very small dictionary containing only five atoms using five face images, by clustering ZCA-whitened patches randomly sampled from the faces, and then take these for feature encoding. Fig. 6 illustrates the face images used for dictionary learning (top) and the five learnt atoms (leftmost). The feature maps of face images encoded by the K-means encoding method and those by the C-SVDD encoding method are respectively shown in Fig. 6 (a) and Fig. 6 (b), where each row are corresponding to one dictionary atom next to it and each column corresponding to one face.

By comparing the feature maps shown in Fig. 6 (a) and Fig. 6 (b), one can see that the C-SVDD-based ones contain more detailed information than the K-means feature maps for the first three atoms, while the responses of the last two atoms are largely suppressed by our method (c.f., last two rows of Fig. 6 (b)). To further understand this phenomenon, we plot the entropy of each atom (by treating them as a small image patch) in Fig. 7 (c). The figure shows that the entropy of the last two atoms is much smaller than that of the first three ones, which indicates that the local appearance patterns captured by these last two atoms are much simpler than those by the first three. Hence these two atoms will tend to be widely used by many faces, resulting in reduced discriminative capability in distinguishing different subjects. In this sense, it will be useful to suppress their responses (c.f., the last two rows of Fig. 6 (b)).

It is also useful to inspect the distribution of local facial patches attracted by these atoms. Fig. 7 (a) gives the results. It can be seen that this distribution is not uniform and the number of local patches attracted by the fourth atom is significantly larger than those by other atoms. As a result, for K-means encoding method, the feature maps yielded by this atom show much more rich details than others (see the fourth row of Fig. 6 (a)), potentially indicating that it could play more important roles than others in the subsequent classification task. However, as explained above, since this atom actually contain much less information than the first three atoms (low
entropy and being a “common word”), it is really not good to over-emphasize its importance in feature encoding.

This drawback of K-means feature mapping is largely bypassed by our C-SVDD-based scheme. As shown Fig.7 (b), the fourth atom actually represents a very small cluster. In fact, the radius of C-SVDD ball corresponding to the more informative atom tends to be large, and one major advantage of our C-SVDD-based strategy is that it is capable to exploit this characteristic of dictionary atoms for more effective feature encoding, as shown in the first three rows of Fig.6 (b). This partially explains the superior performance of the proposed C-SVDD method compared to its K-means counterpart (c.f., experimental results in Section IV).

E. Encoding Feature Maps with SIFT Representation

To encode the feature maps we use a variant of SIFT-representation. SIFT is a widely used descriptor in computer vision and is helpful to suppress the noise and improve the invariant properties of the final feature representation. However, one problem of SIFT-based representation is due to its high dimensionality. For example, if we extract 128 dimensional SIFT-descriptors densely in 250 feature maps with the size of $23 \times 23$ in pixel, the dimension of the obtained representation vector will be as high as over 16M ($250 \times 23 \times 23 \times 128 = 16,928,000$). To address this issue, we first divide each feature map into $m \times m$ blocks and then extract an 8-bit gradient histogram from each block in the same way as SIFT does. This results in a feature representation with dimension of $m \times m \times 8$ for each map, hence significantly reducing the dimensionality while preserving rich information for the subsequent task of pattern classification.

F. Multi-scale Receptive Field Voting

Next we extend our method to exploit multi-scale information for better feature learning. A multi-scale method is a way to describe the objects of interest in different sizes of

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Fig. 6. Illustration of feature maps of five face images (top) using K-means (a) and C-SVDD (b) respectively, based on five dictionary atoms (leftmost), where maps in each row are corresponding to one atom next to it while each column corresponding to one face. For the response values in a feature map, the darker the lower.

Fig. 7. Distribution of the number of patches attracted by each atom (a), the radius of the corresponding SVDD ball (b), and the entropy (c) over the five atoms shown in Fig.6 (rightmost)
context. This would be useful since patches of a fixed size can seldom characterize an object well - actually they can only capture local appearance information limited in that size. For example, if the size is very small, information about edges could be captured but the information on how to combine these into more meaningful patterns such as motifs, parts, poselets, and object, is lost, while information about these entities at different levels is valuable in that they are not only discriminative by itself but complementary to each other as well. Most popular manually designed feature descriptors, such as SIFT or HoG, address this problem to some extend by pooling image gradients into edglets-like features, but it is still unclear, for example, how to assemble edglets into motifs using these methods. Convolutional neural network provides a simple and comprehensive solution to this issue by automatically learn hierarchies of features ranging from edglets to objects. However, during this procedure, information on where those high-level patterns are found becomes more and more ambiguous.

In our work only a single-layer network is used, hence it is difficult to learn multi-scale information in a hierarchical way. Instead, we take a naive way to obtain multi-scale information by using receptive fields of different sizes. In particular, we fetch patches with $S_i \times S_i, i = 1, 2, 3$ squares in size from training images and use these to train dictionary atoms with corresponding size through K-means. Fig.8 shows some examples of atoms we learnt on a face dataset. One can see that these feature extractors are similar to those learnt using a typical ConvNet. Specifically, with the increasing window size, the learnt features become more understandable - for example, as shown in Fig.8 (c), using a receptive field with size of $20 \times 20$ on face images of $64 \times 64$, we successfully learned facial parts such as the eyes, the mouth, and so on, while a smaller receptive field gives us some oriented filters, as shown in Fig.8 (a). At each scale we train several networks with different pooling size. One advantage of this method is that it is very efficient to learn and is effective in capture salient features in a multi-scale context. However, it will not tell us how the bigger patterns are explained by smaller ones - such information would be useful from a generative angle.

To use the learnt multi-scale information for classification, we train a separate classifier on the output layer of the corresponding network (view) according to different receptive sizes and different pooling sizes, then combine them under a boosting framework. Particularly, assume that the total number of categories is $C$, and we have $M$ scales (with $K$ different number of pooling sizes for each scale), then we have to learn $M \times K$ multi-class classifiers. Let us denote the parameter of the $t-$th classifier $\theta_t \in R^{D \times C}$ ($D$ is the dimension of feature representation) as $\theta_t = [w_{t1}, w_{t2}, ..., w_{tC}]$, where $w_{tk}$ is the weight vector for the $k$-th category. We first train these parameters using a series of one-versus-rest $L_2$-SVM classifiers, and then normalize the outputs of each classifier using a soft max function,

$$f_{tk}(x_i) = \frac{\exp(w_{tk}^T x_i)}{\sum_{c=1}^{C} \exp(w_{tc}^T x_i)}. \tag{12}$$

Finally, the normalized predictions $f_{tk}$ are combined to make the final decision,

$$g(x_i) = \arg \max_c \sum_t a_{tc} f_{tk}(x_i). \tag{13}$$

where $f_t = \{f_{t1}, f_{t2}, ..., f_{tC}\}$ is the output vector of the $t$-th classifier, and the corresponding combination coefficients $a_{tc}$ are trained using the following objective,

$$\min_{a_{tc}} \sum_i \max(0, 1 - \sum_t a_{tc}^T f_t(x_i))^2 + \lambda ||a_c||_2 \tag{14}$$

This is the same type of one-versus-rest $L_2$-SVM mentioned before.

IV. EXPERIMENTS AND ANALYSIS

To evaluate the performance of the improved K-means network, we conduct extensive experiments on four object datasets (MINST [2], NORB [24], CIFAR-10 [10], STL-10 [10]). Details about these datasets are given below.

![Fig. 8. Features of different scales learnt from face images. The size of original face images is $64 \times 64$ in pixels. (a) size = 5 (b) size = 10 (c) size = 20](image-url)
**TABLE I**

DEFAULT PARAMETER SETTINGS FOR OUR METHODS.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>#clusters</td>
<td>≤ 500</td>
</tr>
<tr>
<td>Size of receptive field</td>
<td>$5 \times 5, 7 \times 7, 9 \times 9$</td>
</tr>
<tr>
<td>size of average pooling</td>
<td>$4 \times 4, 1 \times 1, 3 \times 3$</td>
</tr>
<tr>
<td>$\lambda$ of C-SVDD</td>
<td>1*, 0.005</td>
</tr>
</tbody>
</table>

*default setting for the non-multi-scale network.

**A. Experiment Settings**

All the images undergo whitening preprocessing before feeding them into the network. The whitening operation linearly transforms the data such that their covariance matrix becomes unit sphere, hence justifying the Euclidean distance we use in the K-means clustering procedure.

Unless otherwise noted, the parameter settings listed in Table I apply to all experiments. The influence of some important parameters, such as the number of filters, will be investigated in more detail in the subsequent sections. For single scale network the receptive field is set to be $5 \times 5$ by default across all the datasets, as recommended in [10], while in multi-scale version, we use receptive fields in three scales, as shown in Table I.

For C-SVDD ball there is a regularization parameter $\lambda$ to set. This parameter allows us to control the amount of noise we are willing to tolerate to. As can be seen from Eq.1, a small $\lambda$ value encourages a tight ball. We set $\lambda = 1$ by default for most datasets except for those with too noisy background are set to 0.005.

Throughout the experiments, we use Coates’ K-means “triangle” encoding method [10] (c.f., Section II-B) as baseline (denoted as ‘K-means’), while its direct counterpart method by simply replacing “triangle” encoding with C-SVDD encoding is denoted as ‘C-SVDD’. Furthermore, we denote the proposed single layer network (c.f., Fig. 3) as ‘C-SVDDNet’, and its multi-scale version as ‘MSRV + C-SVDDNet’. In addition, we re-evaluate the baseline method [10] within the proposed network by replacing its component of C-SVDD with the K-means-based encoding, denoted as ‘K-meansNet’.

**Table II**

COMPARATIVE PERFORMANCE (%) ON THE MINST DATASET.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Error(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep Boltzmann Machines [14] (2009)</td>
<td>0.95</td>
</tr>
<tr>
<td>Convolutional Deep Belief Networks [13] (2009)</td>
<td>0.82</td>
</tr>
<tr>
<td>Multi-column deep neural networks [25] (2012)</td>
<td>0.25</td>
</tr>
<tr>
<td>Network in Network [26] (2013)</td>
<td>0.47</td>
</tr>
<tr>
<td>Maxout Networks [27] (2013)</td>
<td>0.45</td>
</tr>
<tr>
<td>Regularization of neural networks [28] (2013)</td>
<td>0.21</td>
</tr>
<tr>
<td>PCANet [22] (2014)</td>
<td>0.62</td>
</tr>
<tr>
<td>Deeply-Supervised Nets [29] (2014)</td>
<td>0.39</td>
</tr>
<tr>
<td>K-means (1600 features)</td>
<td>1.01</td>
</tr>
<tr>
<td>C-SVDD (1600 features)</td>
<td>0.99</td>
</tr>
<tr>
<td>K-meansNet (400 features)</td>
<td>0.45</td>
</tr>
<tr>
<td>C-SVDDNet (400 features)</td>
<td>0.43</td>
</tr>
<tr>
<td>MSRV+K-meansNet</td>
<td>0.36</td>
</tr>
<tr>
<td>MSRV+C-SVDDNet</td>
<td><strong>0.35</strong></td>
</tr>
</tbody>
</table>

**Table III**

COMPARATIVE PERFORMANCE (%) ON THE NORB DATASET.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Accuracy(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimized ConvNet [3] (2009)</td>
<td>94.4</td>
</tr>
<tr>
<td>Sparse Auto-encoder [10] (2011)</td>
<td>96.9</td>
</tr>
<tr>
<td>Sparse RBM [10] (2011)</td>
<td>96.2</td>
</tr>
<tr>
<td>Regularization of neural networks [28] (2013)</td>
<td>96.77</td>
</tr>
<tr>
<td>K-means (4000 features) [10] (2011)</td>
<td>97.1</td>
</tr>
<tr>
<td>C-SVDD (4000 features)</td>
<td>97.38</td>
</tr>
<tr>
<td>K-meansNet (4000 features)</td>
<td>97.23</td>
</tr>
<tr>
<td>C-SVDDNet (4000 features)</td>
<td>98.18</td>
</tr>
<tr>
<td>MSRV+K-meansNet</td>
<td>97.86</td>
</tr>
<tr>
<td>MSRV+C-SVDDNet</td>
<td><strong>98.64</strong></td>
</tr>
</tbody>
</table>

**B. Experimental results**

In this section we report our experimental results and compare them with some state-of-the-art methods on each dataset. Fig.9 gives sample images of all the datasets used in this work. For each dataset we either use k-fold cross validation or strictly follow the specific evaluation protocol (if available) in order to facilitate comparison with previous work. Note that in all the experiments conducted here, we do not use data augment or data of other datasets.
MINST The MNIST is one of the most popular datasets in pattern recognition. It consists of grey valued images of handwritten digits between 0 and 9 (Fig.9(a)). It has a training set of 60,000 examples, and a test set of 10,000 examples, all of which have been size-normalized and centered in a fixed-size image with 28 × 28 in pixel. In training we use a dictionary with 400 atoms for feature mapping, and after pooling/subsampling we break each feature map into 9 blocks (c.f., Fig. 2 (A)). This results in 6 types of receptive fields (5 × 5, 7 × 7 and 9 × 9). Combined these with two settings for pooling sizes (i.e., 1 × 1 and 2 × 2, respectively), 6 different views/representations can be obtained for each image in this dataset.

Table.II gives our experimental results on the MNIST dataset. It is well-known that deep learning has achieved great success on this task of digit recognition. For example, only 95 among 10,000 test digits are miss-classified by the Deep Boltzmann Machines [14], while Convolutional Deep Belief Networks [13] and Maxout Networks [27] respectively reduce this number to 82 and 45. Our simple single layer network (MSRV+C-SVDDNet) achieves an error as low as 0.35 ± 0.55, which is highly competitive to other complex methods using deep architecture. Fig.10 shows all the 35 misclassified digits by our method, and one can see that these misclassified digits are very confusing even for human beings. Compared to the original K-means network [10], the proposed method reduces the error rate by 65%, with much smaller number of filters.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Accuracy(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum-Product Networks [31] (2012)</td>
<td>83.96</td>
</tr>
<tr>
<td>Learning smooth pooling [32] (2013)</td>
<td>80.02</td>
</tr>
<tr>
<td>Maxout Networks [27] (2013)</td>
<td>88.32</td>
</tr>
<tr>
<td>Stochastic Pooling [33] (2013)</td>
<td>84.87</td>
</tr>
<tr>
<td>Probabilistic Maxout Units [34] (2013)</td>
<td>88.65</td>
</tr>
<tr>
<td>PCANet [22] (2014)</td>
<td>78.67</td>
</tr>
<tr>
<td>Convolutional Kernel Networks [35] (2014)</td>
<td>82.12</td>
</tr>
<tr>
<td>K-means (4000 features) [10] (2011)</td>
<td>79.6</td>
</tr>
<tr>
<td>C-SVDD (4000 features)</td>
<td>79.8</td>
</tr>
<tr>
<td>K-meansNet (1200 features)</td>
<td>82.23</td>
</tr>
<tr>
<td>C-SVDDNet (1200 features)</td>
<td>82.64</td>
</tr>
<tr>
<td>MSRV+K-meansNet</td>
<td>84.96</td>
</tr>
<tr>
<td>MSRV+C-SVDDNet</td>
<td>85.30</td>
</tr>
</tbody>
</table>

This reveals that at least on this dataset with clean background, it is very beneficial to focus more on the representation of the details of the image, rather than emphasizing too much on its global aspects using a large number of filters and a large pooling size (c.f., Fig. 2 (A)).

NORB The dataset contains large amounts of object images in five classes including animals, humans, planes, trucks, and cars (c.f., Fig.9(b)). The dataset comes in two different versions, normalized-uniform and jittered-cluttered. In this paper we use the normalized-uniform version. The image size is 96 × 96 in pixel and objects are centred in the images with a uniform background. There are 10 instances of each object class, imaged under 6 illuminations and 162 viewpoints (18 azimuths 9 elevations). According to the evaluation protocol, these instances are split into two equal-size disjoint sets containing 29,160 images each, used for training and testing respectively. At test time a trained model has to recognize unseen instances of the same object classes. We resize all images to 64 × 64 in pixel and use average pooling with a block size of 4 × 4. Finally, a 72-dim SIFT representation is extracted (from 9 blocks) for each feature map. For multi-scale receptive voting, we use 2 types of receptive fields (5 × 5 and 7 × 7) and 3 types of pooling sizes (2 × 2, 3 × 3 and 4 × 4). This results in 6 different views/representations for each input image.

Table.III gives the results. One can see from the table that our method achieves the best result among the compared methods with an accuracy of 98.64%. Note that this is a huge dataset containing nearly 60 thousands data with both viewpoint and lighting variations (c.f., Fig.9(b)) and many feature learning algorithms use this database as a benchmark to evaluate their scalability, such ConvNet [24] [3], DBM [14], DBN [30], Locally-connected Neural Pyramid (LCNP) [17] and many sparse coding methods [10] [10]. It is previously shown that a fine-tuned K-means single-layer network [10] yields better performance than those aforementioned methods.
However, we achieve the state of the art performance with only 400 atoms, about 10 times smaller than that used in [10].

**CIFAR-10** The CIFAR-10 dataset consists of 60,000 $32 \times 32$ colour images from 10 classes, with 6,000 images per class. According to the evaluation protocol, 50,000 images are used for training and the remaining 10,000 images are used for testing. Note that images in CIFAR-10 vary significantly not only in object positions and object scales within each class, but also in colors and textures (c.f., Fig.9(c)). Furthermore, many images are with cluttered background and in low resolution. All these factors impose great challenges to our algorithm. To avoid bring too much noise into the representation, we focus more on the global aspects of the image when feature learning, by using a large dictionary with 1,200 atoms, two small pooling sizes (1 $\times$ 1 and 2 $\times$ 2,), and 3 different receptive scales (5 $\times$ 5, 7 $\times$ 7, and 9 $\times$ 9). The feature map is divided into 4 blocks, yielding 32-bit SIFT representation for each map. We also set a small $\lambda$ value in the C-SVDD training (c.f., Eq. 6) to encourage larger SVDD balls.

Table IV gives the results. It shows that a single scale C-SVDDnet achieves an accuracy of 82.64%, which is comparable or superior to the performance of other multi-layer architecture systems, such as PCANet [22], Convolutional Kernel Networks [35] and [10]. One possible explanation is that our compact SIFT encoding leads to more robust representation than others, such as Learning smooth pooling [32] and Sum-Product Networks [31]. By integrating multi-scale information we improve the accuracy to 85.30%, close to the state of the art performance on this benchmark.

**STL-10** The STL-10 is a large image dataset popularly used to evaluate algorithms of unsupervised feature learning or self-taught learning (c.f., Fig.9(d)). Besides 100,000 unlabeled images, it contains 13,000 labeled images from 10 object classes, among which 5,000 images are partitioned for training while the remaining 8,000 images for testing. All the images are color images with 96 $\times$ 96 pixels in size. There are 10 predefined overlapped folds of training images, with 1000 images in each fold. In each fold, a classifier is trained on a set of 1000 training images, and tested on all 8000 testing images. In consistence with [10], we report the average accuracy across 10 folds. For unsupervised feature learning we randomly select 20,000 unlabeled data. The size of spatial pooling is 4 $\times$ 4, hence the size of feature maps fed for SIFT representation is 23 $\times$ 23. For multi-scale receptive voting we use 2 scale (5 $\times$ 5 and 7 $\times$ 7), on each of which we perform spatial pooling in 5 sizes ranging from 2 $\times$ 2 to 6 $\times$ 6.

Table V gives our results on the STL-10 dataset. The major challenges of this dataset lie in that its images are captured in the wild with cluttered background, objects in various scales and poses (c.f., Fig.9(d)). As before, we compared our method with several feature learning methods with state of the art performance. One can see that our one scale C-SVDD network obtains 65.62% accuracy, using a filtering dictionary of 500 atoms, better than 58.6% of Trans. Invariant RBM (TIRBM) [37], 60.10% of Selective Receptive Fields (SRF) [36], and 62.30% of Discriminative Sum-Product Networks (DSPN) [31]. Also note that replacing the proposed C-SVDD encoding with K-means encoding leads to over 3.0% performance loss, while fusing the multi-scale information gives us about 2.6% improvement in accuracy, exceeding the current best performer [39] on this challenging dataset.

C. The Influence of Parameters and Discussions

In this section, we investigate the influence of several key parameters of our method on the performance, and empirically illustrate the contributions of the individual stage of our unsupervised feature learning chain. This series of experiments are performed on the STL-10 dataset.

**Impact of the number of features** By the number of features, we mean the number of filters $K$ used for feature extraction, which is equal to the number of dictionary atoms. Fig.11 gives the performance curves according to varying number of features with different methods on the STL-10 dataset. It can be seen that with the increasing number of features, the performance of both K-means and C-SVDD methods rises, which is consistent with the results by Coates et al. [10]. One possible explanation is that since both K-means encoding and C-SVDD encoding use the learnt dictionary to extract non-linear features, more dictionary atoms help to disentangle factors of variations in images. In our opinion the capability to learn a large number of atoms at relatively low computational cost is one of the major advantages of K-means based methods for unsupervised feature learning over other algorithms such as Gaussian Mixture Model (GMM), sparse coding, and RBM. For example, it is difficult for a GMM to learn a dictionary with over 800 atoms [10].

On the other hand, a too large dictionary can increase the redundancy and decrease the efficiency. Hence it is desirable to reduce the number of features while not hurting the performance too much. Fig.11 shows that our C-SVDD encoding method consistently works better than the K-means encoding at different number of features, and combining C-SVDD encoding and SIFT-based representation dramatically reduces the needs for large dictionary without scarifying the
Fig. 12. The effect of different pooling sizes on the performance with the proposed method on the STL-10 dataset.

To investigate the effect of different pooling sizes on the performance using the proposed method, we conduct a series of experiments on the STL-10 dataset. Particularly, for a 96 × 96 original image, we use a receptive field of 5 × 5 in pixel for feature extraction and obtain a layer of feature maps with 92 × 92. The pooling blocks are set to be \(m \times m\) such that the size of final feature maps after pooling is \(92/m \times 92/m\). We vary \(m \times m\) from 1 × 1 to 31 × 31 and record the yielded accuracy. Fig.12 gives the results under different settings. We can see from the figure that generally for the one layer K-means-based network we need bigger block sizes for improved translation invariance, but adding a robust SIFT encoding layer after pooling effectively reduces the needs for large pooling size while obtaining better performance. One possible reason is that this tends to characterize more detailed information of the objects to be represented.

Effect of the multi-scale receptive field voting Fig.13 gives the detailed accuracy of 10 representations using 2 sizes of receptive fields and 5 sizes of pooling blocks. One can see that different representation leads to different prediction accuracy but combining them leads to better performance. This shows that the representations captured with different receptive fields and pooling sizes are complementary to each other.

Contribution of components To illustrate the contributions of the individual stages of the proposed method (i.e., C-SVDD-based encoding, SIFT-representation and multi-scale voting), we conduct a series of experiments on the STL-10 dataset by removing each of the three main stages in turn while leaving the remaining stages in place (the comparison is thus against our full method). Fig.14 gives the results. In general each stage is beneficial and (not shown) the results are cumulative over the stages, but the SIFT stage seems to contribute most to the performance improvement.

V. CONCLUSION

In this paper, we propose a simple one-layer neural network termed C-SVDDNet for unsupervised feature learning. The key idea of the proposed method is to describe the density and distribution of each cluster from K-means with an SVDD ball for more robust feature representation. For this purpose, we present a new SVDD algorithm called C-SVDD that centers the SVDD ball towards the mode of local density of each cluster. Furthermore we show that the objective of C-SVDD can be solved very efficiently as a linear programming problem. Another key component of the C-SVDDNet is a post-pooling layer, which effectively encodes the pooling responses with a variant SIFT descriptors. With the help of this, we can characterize the details of the input without needing to learn a large number of filters, hence allowing a “light” design strategy for network architecture. Last but not least, we extend
the network with multiple receptive field scales and multiple pooling sizes for better feature learning. Extensive experiments on several popular object recognition benchmarks demonstrate that the proposed C-SVDDNet yields comparable or better performance than that of the previous state of the art methods.

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