Two-Dimensional Bayesian Subspace Analysis for Face Recognition

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Abstract. Bayesian subspace analysis (BSA) has been successfully applied in data mining and pattern recognition. However, due to the use of probabilistic measure of similarity, it often needs much more projective vectors for better performance, which makes the compression ratio very low. In this paper, we propose a novel 2D Bayesian subspace analysis (2D-BSA) method for face recognition at high compression ratios. The main difference between the proposed 2D-BSA and BSA is that the former adopts a new Image-as-Matrix representation for face images, opposed to the Image-as-Vector representation in original BSA. Based on the new representation, 2D-BSA seeks two coupled set of projective vectors corresponding to the rows and columns of the difference face images, and then use them for dimensionality reduction. Experimental results on ORL and Yale face databases show that 2D-BSA is much more appropriate than BSA in recognizing faces at high compression ratios.

1 Introduction

Subspace analysis has attracted much attention in machine learning, data mining and pattern recognition over the last decade. The essence of subspace analysis is to find a set of projective vectors, through which to represent original high-dimensional faces in a low-dimensional space. Principal component analysis (PCA, also known as Eigenface) [5], linear discriminant analysis (LDA) [1][2] and the Bayesian subspace analysis (BSA) [3] [4] are the three mainstreams of subspace analysis methods in the field. Among them, PCA does not consider the class label information and hence is unsupervised. LDA uses the class label information but its decision boundaries are crisp and simple (linear) in nature. The BSA method also uses supervised information, but in a way different from LDA, that is, it tries to construct the similarity model (i.e., intrapersonal space) of the same individual in a soft (probabilistic) way. This makes it easier to adapt to unknown samples. And it has been shown that BSA outperforms both PCA and LDA [4].

However, one of the limitations of BSA is that it needs relatively more projective vectors to compute the probabilistic measure of similarity for better performance, which makes the compression ratio very low. Here the compression ratio is defined as the division between the total numbers of training face image pixels and the projected face components plus the sizes of projective vectors. More specifically, suppose there are *M* training face images, each of size *m* by *n*, and then the total numbers of training face image pixels is *Mmn*. If *d* projective vectors are used in BSA, the size of projective vectors is *dmn*, the numbers of projected face components is *Md*, and then the compression ration is computed as (*Mmn*)/(*Md*+*dmn*). Clearly, as the number of projective vectors increases, the compression ratio reduces. On the other hand, as the number of projective vectors increases, the size of the projected face vector also adds, which means the processing time for recognizing faces also increases. That limitation is especially severe on large face databases.

In this paper, we propose a novel 2D Bayesian subspace analysis (2D-BSA) method for face recognition at high compression ratios. Opposed to the classical Image-as-Vector representation in original BSA, we adopt a new Image-as-Matrix representation [6] for face images in 2D-BSA. Based on the new representation, 2D-BSA seeks two set of projective vectors corresponding to the rows and columns of the difference face images, and then use them for dimensionality reduction. According to the type of projective vectors used, 2D-BSA divides into three concrete forms, i.e. unilateral 2D-BSA using only the projective vectors corresponding to the rows or columns of the difference face images (denoted as U2D-BSA-row and U2D-BSA-col respectively) and bilateral 2D-BSA using both set of projective vectors (denoted as B2D-BSA). Experimental results on ORL and Yale face databases show that the proposed 2D-BSAs (especially B2D-BSA) outperform original BSA in recognizing faces at high compression ratios.

The rest of this paper is organized as follows: In Section 2, we briefly introduce the original BSA method. And we propose the 2D-BSA methods in Section 3. The experimental results on ORL and Yale face databases are given in Section 4. Finally, we conclude in Section 5.

2 Bayesian Subspace Analysis

The main idea of Bayesian subspace analysis lies in developing a probabilistic measure of similarity based on a Bayesian (MAP) analysis of face image differences. Consider a feature space of the differences vectors $\Delta = I_1 - I_2$ between two images. Define two classes of facial image variations: intrapersonal variations Ω_I (corresponding to different facial expressions of the same individual) and extra-personal variations Ω_E (corresponding to variations between different individuals). The similarity measure $S(\Delta)$ can then be expressed in terms of the intrapersonal a posteriori probability of Δ belong to Ω_I given by the Bayesian rule [4]:

$$S(\Delta) = P(\Omega_I \mid \Delta) = \frac{P(\Delta \mid \Omega_I) P(\Omega_I)}{P(\Delta \mid \Omega_I) P(\Omega_I) + P(\Delta \mid \Omega_E) P(\Omega_E)} \tag{1}$$

The densities of both classes are modeled as high-dimensional Gaussians [4]:

$$P\left(\Delta \mid \Omega_E\right) = \frac{e^{-\frac{1}{2}\Delta^T \Sigma_E^{-1} \Delta}}{(2\pi)^{D/2} \left|\Sigma_E\right|^{1/2}}, \quad P\left(\Delta \mid \Omega_I\right) = \frac{e^{-\frac{1}{2}\Delta^T \Sigma_I^{-1} \Delta}}{(2\pi)^{D/2} \left|\Sigma_I\right|^{1/2}} \qquad \textbf{(2)}$$

where Σ_E and Σ_I are the covariance matrices of Ω_E and Ω_I respectively.

To compute the likelihoods $P\left(\Delta \mid \Omega_E\right)$ and $P\left(\Delta \mid \Omega_I\right)$, the database images I_j are preprocessed with the whitening transformation. Each image is converted and stored as a set of two whitened subspace coefficients, i.e. $y_{\phi_I}^j$ for intrapersonal space and $y_{\phi_E}^j$ for extrapersonal space [4]:

$$y_{\phi_I}^j = \Lambda_I^{-\frac{1}{2}} V_I I_j, \qquad y_{\phi_E}^j = \Lambda_E^{-\frac{1}{2}} V_E I_j \qquad \textbf{(3)}$$

where Λ_I, V_I and Λ_E, V_E are matrices of the largest eigenvalues and corresponding eigenvectors of the covariance matrices of Σ_I and Σ_E respectively.

From Eq. (3), Eq. (2) can be rewritten as [4]:

$$P\left(\Delta \mid \Omega_E\right) = \frac{e^{-\left\| y_{\phi_E} - y_{\phi_E}^j \right\|}}{(2\pi)^{k_E/2} \left|\Sigma_E\right|^{1/2}}, \quad P\left(\Delta \mid \Omega_I\right) = \frac{e^{-\left\| y_{\phi_I} - y_{\phi_I}^j \right\|}}{(2\pi)^{k_I/2} \left|\Sigma_I\right|^{1/2}} \qquad \textbf{(4)}$$

where k_E and k_I are the reduced dimensions of Ω_E and Ω_I respectively, and $y_{\phi E}$ and $y_{\phi I}$ are the whitened coefficient vectors for the test image *I*.

From Eq. (4), the maximum a posteriori (MAP) similarity defined in Eq. (1) can be easily computed. However, in practice, the MAP similarity is often replaced with the following maximum likelihood (ML) similarity [4]:

$$S'\left(\Delta\right) = P\left(\Delta \mid \Omega_I\right) = \frac{e^{-\left\| y_{\phi_I} - y_{\phi_I}^j \right\|}}{(2\pi)^{k_I/2} \left|\Sigma_I\right|^{1/2}} \qquad \textbf{(5)}$$

In Eq. (5), only the intrapersonal class is evaluated, and it has been shown that the ML similarity measure has a similar performance to that of the MAP similarity measure. For that reason and for simplicity, we only consider the ML similarity measure defined in Eq. (5) throughout the paper.

3 2D Bayesian Subspace Analysis

Suppose that there are M training face images, denoted by m by n matrices $A_k (k = 1, 2, ...M)$. Let $\Omega_I = \{B_i\}_{i=1}^{N}$ denote the set of difference images from the same individual. Concatenating N matrices B_i into an m by nN matrix:

$$M_L = [B_1, B_2, ..., B_N] = \{b_j\}_{j=1}^{nN} \tag{6}$$

where b_j s is the m by 1 column vectors of B_i s.

Let Λ_{Ld} (d by d diagonal matrix) and $L = [l_1, l_2, ..., l_d]$ (n by d matrix) be the d largest eigenvalues and corresponding eigenvectors of Eq. (6). For each training image A_k and any test image A, the column whitening transformation is as follows:

$$y_{\phi_L}^k = \Lambda_{Ld}^{-\frac{1}{2}} L^T A_k, \quad y_{\phi_L} = \Lambda_{Ld}^{-\frac{1}{2}} L^T A \tag{7}$$

From Eq. (7), the ML similarity becomes:

$$S_L(\Delta = A - A_k) = \frac{e^{-\left\| y_{\phi_L} - y_{\phi_L}^k \right\|}}{(2\pi)^{d/2} |\Sigma_L|^{1/2}} \tag{8}$$

In this paper, we call the 2D-BSA method based on Eq. (8) as unilateral 2D-BSA with column whitening, denoted as U2D-BSA-col.

Similarly, Concatenating N matrices B_i into an mN by n matrix:

$$M_R = \left[B_1^T, B_2^T, ..., B_N^T\right]^T = \{c_j\}_{j=1}^{mN} \tag{9}$$

where c_j s is the 1 by n row vectors of B_i s.

Let Λ_{Rd} (d by d diagonal matrix) and $R = [r_1, r_2, ..., r_d]$ (m by d matrix) be the d largest eigenvalues and corresponding eigenvectors of Eq. (9). For each training image A_k and any test image A, the row whitening transformation is as follows:

$$y_{\phi_R}^k = A_k R \Lambda_{Rd}^{-\frac{1}{2}}, \quad y_{\phi_R} = A R \Lambda_{Rd}^{-\frac{1}{2}} \tag{10}$$

And the ML similarity becomes:

$$S_R(\Delta = A - A_k) = \frac{e^{-\left\| y_{\phi R} - y_{\phi R}^k \right\|}}{(2\pi)^{d/2} |\Sigma_R|^{1/2}} \tag{11}$$

We call the 2D-BSA method based on Eq. (11) as unilateral 2D-BSA with row whitening, denoted as U2D-BSA-row.

Finally, if we have obtained the aforementioned Λ_{Ld} , $L = \left[l_1, l_2, ..., l_d \right]$, and Λ_{Rd} , $R = \left[r_1, r_2, ..., r_d \right]$. For each training image A_k and any test image A, we can define the following bilateral whitening transformation:

$$y_{\phi_B}^k = \Lambda_{Ld}^{-\frac{1}{2}} L^T A_k R \Lambda_{Rd}^{-\frac{1}{2}}, \quad y_{\phi_B} = \Lambda_{Ld}^{-\frac{1}{2}} L^T A R \Lambda_{Rd}^{-\frac{1}{2}} \tag{12}$$

And the ML similarity measure is computed as:

$$S_B \left(\Delta = A - A_k \right) = \frac{e^{-\left\| y_{\phi B} - y_{\phi B}^k \right\|}}{(2\pi)^{d/2} \left| \Sigma \right|^{1/2}} \tag{13}$$

And we call the 2D-BSA method based on Eq. (13) as bilateral 2D-BSA, denoted as B2D-BSA.

4 Experimental Result

In this section, we test the proposed U2D-BSA-col, U2D-BSA-row and B2D-BSA methods, compared with original BSA method, on two commonly used face databases, ORL and Yale face database. The ORL database contains images from 40 individuals, each providing 10 different images. The Yale database contains images from 15 individuals, each providing 11 different images. For both database, the first 5 images per person are used for training, and the rest for testing. Here the well-know nearest neighbor (1-NN) classifier is used for classification, after extracting the features using the above mentioned methods.

Figure 1 and Fig. 2 show the comparisons of the recognition accuracy under different compression ratios of the four methods on ORL and Yale face databases respectively. Here the compression ratio is defined as the division between the total numbers of training face image pixels and the projected face components plus the sizes of projective vectors. More specifically, the compression ratios of BSA, U2D-BSA-col, U2D-BSA-row and B2D-BSA are (Mmn)/(Md+dmn), (Mmn)/(Mdn+dm), (Mmn)/(Mdm+dn) and (Mmn)/(Md^2+dm+dn) respectively, where M is the number of training images, m and n are the size of the face image, and d is the number of projective vectors. Note that M, m and n are fixed for certain face database, and the compression ratio is directly related with only the reduced dimensions d.

From Fig. 1 and Fig. 2, it is impressive to see that B2D-BSA has the best accuracy no matter what compression ratio is chosen. Both U2D-BSA-col and U2D-BSA-row outperforms BSA, but they are inferior to B2D-BSA. As the compression ratio increases, the accuracy of BSA decreases rapidly. On the other hand, B2D-BSA can still retain a relatively high accuracy even when the compression ratio is larger than 100.

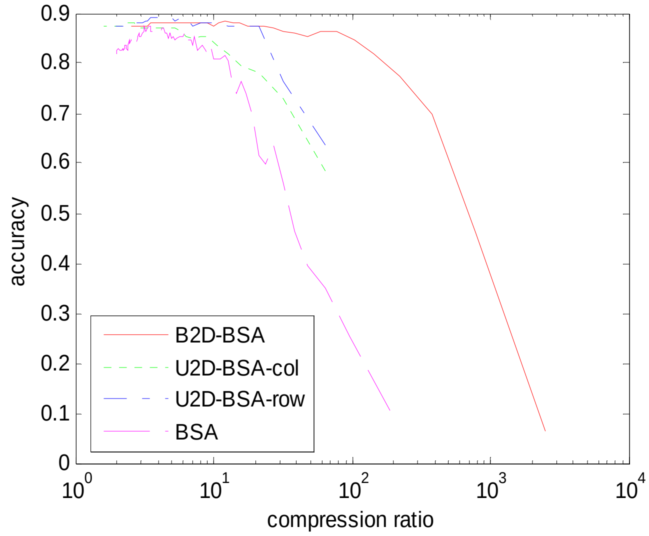

Fig. 1. Comparisons of the recognition accuracy under different compression ratios of the four methods on ORL face database.

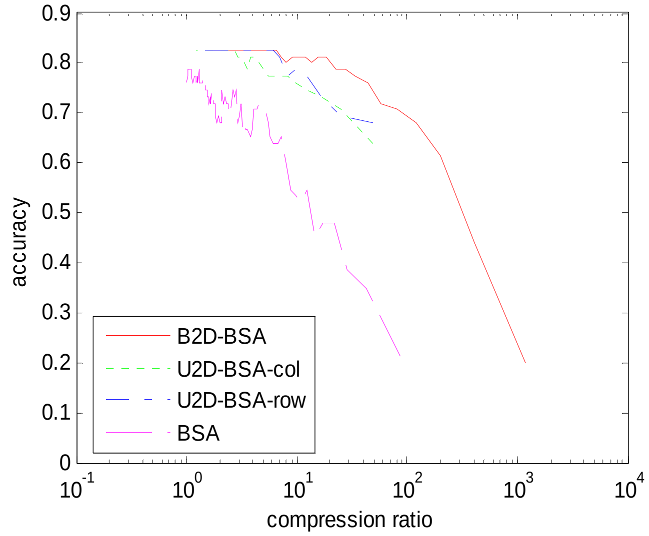

Fig. 2. Comparisons of the recognition accuracy under different compression ratios of the four methods on Yale face database.

5 Conclusion

In this paper, we propose a novel 2D Bayesian subspace analysis (2D-BSA) method for face recognition at high compression ratios. The main difference between the proposed 2D-BSA and BSA is that the former adopts a new Image-as-Matrix representation for face images, opposed to the Image-as-Vector representation in original BSA. Experimental results show the effects of the proposed method.

Both BSA and the proposed 2D-BSA methods assume the Gaussian likelihood function which is not usual in real data. So an interesting issue is to extend the proposed methods to non-Gaussian case though some kernel transformations [7]. Also, in the future works, we will compare our methods with existing face recognition methods such as Eigenface [5] and Fisherface [1] etc.

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