A novel multi-valued BAM model with improved error-correcting capability *

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Abstract A Hyperbolic Tangent multi-valued Bi-directional Associative Memory (HTBAM) model is proposed in this letter. Two general energy functions are defined to prove the stability of one class of multi-valued BAMs, with HTBAM being the special case. Simulation results show that HTBAM has a competitive storage capacity and much more error-correcting capability than other multi-valued BAMs.

Keywords bi-directional associative memory, recurrent neural network, multi-value

I. Introduction

The Bi-directional Associative Memory (BAM) proposed by B. Kosko [1] is an extension of the Hopfield memory from auto-association to bi-directional association. Owing to its good generalization and noise immunity, BAM is well suited for patter recognition. However, the original Kosko's BAM suffers from low storage capacity and recall reliability. Many efforts have been invested to improve the performance of Kosko's BAM [2-12]. These works can be roughly classified into two groups. In the first group, the data representation is still limited to bipolar or binary vectors and the tasks are focused on either adding dummy neurons, more layers or interconnections among neurons inside each layer, or introducing new learning algorithms to

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improve the performance. Most works belong to the first group [2-5, 7, 8, 12, 13]. However, another method to enlarge the capacity is to expanse the data range, i.e., from binary value to multi-value. With the multi-value idea borrowed from the Multi-Valued Exponential Correlation Associative Memory (MV-ECAM) [13], Wang proposed the Multi-Valued exponential Bi-directional Associative Memory (MV-eBAM) [9], which was based on the exponential generating function. A modified MV-eBAM was also introduced by replacing the 2-norm in the exponent with 1-norm, and an extended model introducing interconnections among neurons inside each layer was proposed by Chen [6]. Recently, Wang presented another multi-valued BAM using the polynomial as the generating function (PBHC) [10]. Although it was shown that PBHC had higher storage capacity than other BAMs, its error-correcting capability was not mentioned in the original paper. In this letter, we proposed another type of multi-valued BAM, adopting the hyperbolic tangent as the generating function (HTBAM). And a general method for proof of stability is presented which can be applied into one class of multi-valued BAMs. Furthermore, we find an interesting relationship between the above-mentioned BAMs and the kernel method in statistical learning theory. In fact, the generating functions used in PBHC, MV-eBAM and HTBAM respectively are the three most commonly used kernels, i.e., polynomial, radial-basis function, and neural network type. Thus we have the theoretical basis for choosing generating functions. Experimental results show that HTBAM have competitive storage capacity and much higher error-correcting capability than other multi-valued BAMs.

II. Proposed HTBAM model

Suppose we are given M stored pattern pairs (X_i , Y_i), i=*1, 2, ..., M*, where $X_i \in \{1,2,\ldots,L\}^n$, and $Y_i \in \{1,2,\ldots,L\}^p$. Instead of using Kosko's approach, we use the following evolution equations

$$X = H\left(\frac{\sum_{i=1}^{M} X_i (1+\tanh(-\| Y_i - Y \|^2))}{\sum_{i=1}^{M} (1+\tanh(-\| Y_i - Y \|^2))}\right) \quad (1)$$

$$Y = H\left(\frac{\sum_{i=1}^{M} Y_i (1+\tanh(-\| X_i - X \|^2))}{\sum_{i=1}^{M} (1+\tanh(-\| X_i - X \|^2))}\right) \quad (2)$$

where X and Y are input key patterns, and $H(\cdot)$ is a staircase function shown as the following equation:

$$H(x) = \begin{cases} 1, & x < 1 \\ L, & x > D \\ \left\lfloor \frac{L}{D} x + 0.5 \right\rfloor, & \text{otherwise} \end{cases} \quad (3)$$

where L is the number of finite levels and D is the finite interval of the staircase function.

We discuss the stability of HTBAM. Define two general energy functions from X to Y and from Y to X, respectively, as follows:

$$E_1(X,Y) = \sum_{i=1}^{M} \int_0^{d(X,X_i)} S(u, d(Y,Y_i))du \quad (4)$$

$$E_2(X,Y) = \sum_{i=1}^{M} \int_0^{d(Y,Y_i)} S(d(X,X_i), v)dv \quad (5)$$

where $d(\cdot,\cdot)$ denotes the measure, $S(\cdot,\cdot)$ is the generating function. In HTBAM model, we have $d(x,y) = \| x - y \|^2$, $u = 0, v = 0$ and

$$S(d(x,y)) = 1 + \tanh(-d(x,y)) \quad (6)$$

Suppose X' is the next state of X, then the change of E_1 is

$$\begin{aligned} \Delta_X E_1(X,Y) &= (\nabla_X E_1(X,Y))^T \cdot \Delta X \\ &= -2\left(\sum_{i=1}^{M} S(d(Y_i,Y))\right)\sum_{k=1}^{n}\left[\left(\frac{\sum_{i=1}^{M} x_{ik} S(d(Y_i,Y))}{\sum_{i=1}^{M} S(d(Y_i,Y))} - x_k\right)(x_k' - x_k)\right] \end{aligned} \quad (7)$$

where x_k, x_k' and x_{ik} are the *k*-th component of X, X' and X_i, respectively. Let

$\sigma = \dfrac{\sum_{i=1}^{M} x_{ik} S(d(Y_i, Y))}{\sum_{i=1}^{M} S(d(Y_i, Y))} - x_k$. Thus according to Eqs.(1) and (2), when $-\frac{1}{2} \le \sigma < \frac{1}{2}$, $x_k' = x_k$, $\sigma(x_k' - x_k) = 0$; when $\sigma < -\frac{1}{2}$, $x_k' < x_k$, $\sigma(x_k' - x_k) > 0$; when $\sigma > \frac{1}{2}$, $x_k' > x_k$, $\sigma(x_k' - x_k) > 0$. Since $0 < S(\cdot)$, we always have $\Delta_X E_1(X, Y) \le 0$. So the stability of the $X \to Y$ phase of the network is proved. By following a similar way using Eqs.(5), we can prove the stability of the $Y \to X$ phase.

The above process of proof of stability is a general method for models with the generating function $0 < S(\cdot)$. If we define generating function $S(\cdot)$ as the following equations

$$S(d(x, y)) = b^{-d(x,y)} \tag{8}$$

or

$$S(d(x, y)) = (1 - d(x, y)/u)^{M^z} \tag{9}$$

where $u = \sum_{i=1}^{M}\sum_{j=1}^{M}(\| X_i - X_j \|^2 + \| Y_i - Y_j \|^2)$, b, z are constants, $d(x, y) = \| x - y \|^2$, then we get MV-eBAM and PBHC immediately. For modified MV-eBAM, it uses the measure $d(x, y) = \sum_i | x_i - y_i |$ for Eqs.(8). An interesting thing is that Eqs.(6), (8) and (9) are variations of three kernels used in statistical learning theory, i.e., neural network type, radial basis function and polynomial, respectively. In fact, polynomial function is a global function, radial basis function or Gauss function is a local one, and the neural network type function or hyperbolic tangent function is a tradeoff between global and local, so it can have better noise immunity capability than polynomial function, without loss of some global information.

III. Simulation results

Computer simulations are performed to compare the storage capacity and error correcting capability of HTBAM with MV-eBAM, modified MV-eBAM and PBHC models. In the following

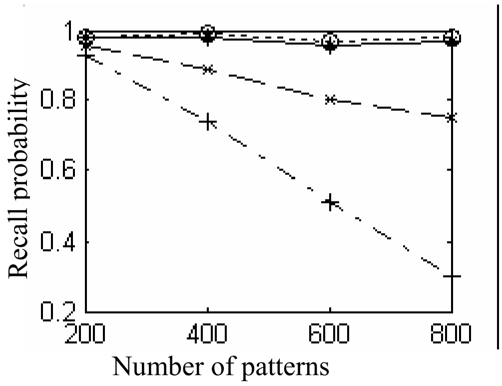

Fig.1 Storage capacity comparison

× MV-eBAM ($b = 2$)

+ modified MV-eBAM ($b = 2$)

○ PBHC ($z = 4$)

* HTBAM

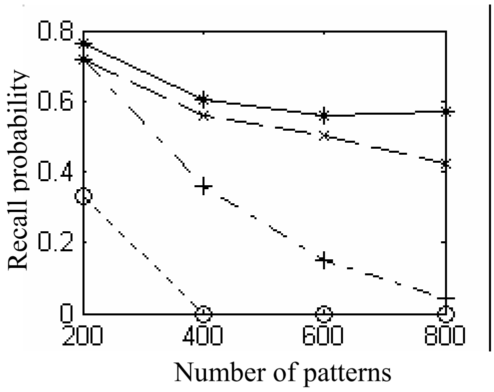

Fig.2 Error-correcting ability comparison

× MV-eBAM ($b = 2$)

+ modified MV-eBAM ($b = 2$)

○ PBHC ($z = 4$)

* HTBAM

experiments we assume $L = D = 8$, and $n = p = 8$. Fig.1 and Fig.2 show the storage capacity and error-correcting capability of the four multi-valued BAMs, respectively. Results are obtained from ten sets of randomly selected associative pattern pairs. For the error-correcting simulation, we randomly change one digit to others in the input pattern. We can see from the above figures that HTBAM has competitive storage capacity with PBHC, and is superior to MV-eBAM and modified MV-eBAM. And the error-correcting capability of HTBAM is the best of the four models, while PBHC has the poorest performance. We also do simulations for PBHC with the parameter $z = 2$. In this situation, PBHC lose its associative ability, i.e., the storage capacity is zero and has no error-correcting capability any more.

IV. Conclusions

An improved multi-valued BAM model has been presented. This model has an increased error-correcting capability, without scarifying its storage capacity. Meanwhile, there is no free parameter in this model to be properly adjusted. Experimental results confirm the effectiveness of the proposed model.

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